ANALYSIS AND APPLICATION OF MAGNETOSTRICTION DELAY LINES

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Abstract.—Uniformly variable sonic delay lines are discussed with particular attention to the factors affecting the formation of the mechanical signal of the transmitting transducer and the electrical signal of the receiving transducer. Examples of pulse shapes for different conditions are calculated and sketched. For comparison with these some experimental results are described and illustrated. Consideration is given to variation of delay, use of multiple transducers, and the control of the polarity of the response. Several devices in which magnetostriction delay lines may be used are briefly described.

I. INTRODUCTION

The extensive use of intentional delay of signals has developed largely since 19__ beginning in radar and other military equipment and found now in many additional applications. Purely electrical means of delay become impractical for delays longer than a few microseconds because of the long paths required at the high velocity of electric wave propagation. Ultrasonic systems, requiring less space for the lower velocities of mechanical wave propagation and correspondingly shorter paths, are firmly established where the longer delays are required. The wide extent of the literature of ultrasonic delay lines has been shown in the 1954 bibliography compiled by M. D. Fagan listing seventy-eight titles.[1] The attention has been given almost exclusively to systems other than magnetostriction types. This has been due mainly to the frequency limitation imposed by the use of the latter.

Three groups of researchers have published papers dealing with magnetostriction delay lines. The earliest paper, by E. M. Bradburd, appeared in 1951, describing lines used as components of aerial navigation equipment.[2] R. Millership, R. C. Robbins, and A. E. DeBarr have described applications to digital computers made in England.[3,4,5] The laboratory note of H. Epstein and O. Stram was published in 1953.[6] It is evident that in some instances factors such as greater reliability and ruggedness, ease of delay variation, saving in cost, space, and weight, as well as other advantages will dictate the use of magnetostriction delay lines in spite of their shortcomings in other respects.

II. THE MECHANICAL SIGNAL FOR STEP INPUT

The basic elements of the line are the transmitting transducer, the delay medium, and receiving transducer. The transmitting transducer transforms the electric signal into a mechanical wave which, after passage down the delay medium, is reconverted in the receiving transducer to an electric signal. Thus if a step of current is applied to the coil a mechanical disturbance will be launched

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in both directions along the axis of the line. Damping must be provided at the ends of the line to eliminate the disturbances caused by reflections. The configuration resulting when the magnitude of the strain is plotted against distance along the longitudinal axis of the line is designated here as the shape of the mechanical pulse. It is known that for a given magnetostrictive medium the shape will be determined by three principal factors: the effective length of the transducer coil, l_1; the time, T, required for the flux in the medium to reach a stable value; and the strength of the magnetic fields applied.[5]

This can be demonstrated in a simple way by considering incremental changes of the flux density occurring at short time intervals, Δt, from 0 to T. At each of the instants, $n\Delta t$ up through time T, an incremental deformation l_1 in length will be launched along the line at velocity c. The shape resulting from the summation of these incremental strain pulses is shown in Fig. 1. The width of this disturbance, which is outlined with the heavy line, is seen to be the sum of the transducer length plus the product Tc.
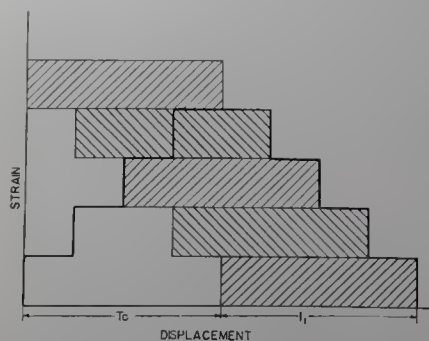

Fig. 1—*The strain pulse formed in the medium by the transmitting transducer is represented by the heavy outline. The shaded rectangles represent the strain increments which are summed to get the pulse.*
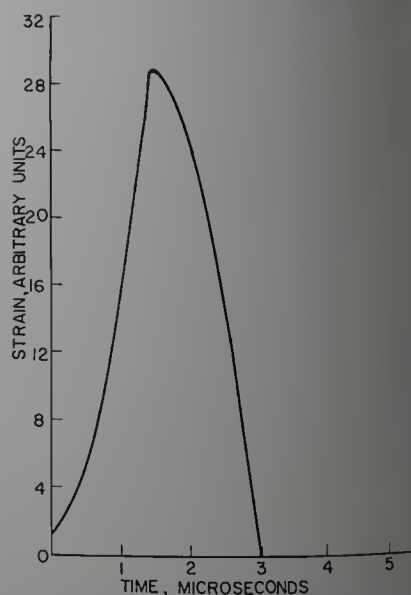

Fig. 2 —*Configuration representing response of transmitter to a step input, with strain proportional to the square of the flux density.*

For low flux densities the deformation will be proportional to the square of the flux density.[7] Thus

(1) $$\delta = \alpha B^2$$

where δ is the strain.

Since B is a function of time,

(2) $$\delta(t) = \alpha f(t)$$

From (1) it can be seen that a unit step of B will produce a strain pulse of intensity α and l_1 in length. This response, the analog of the indicial admittance of an electrical network, can be expressed as a pulse

$$(3) \qquad \alpha \left(S(t-t_0) - S(t-t_0 - \frac{l_1}{c}) \right)$$

where $S(t-t_0)$ represents a unit step function at t_0. Applications of the superposition theorem will yield an expression for strain as a function of time. From this the shape of the mechanical pulse may be determined.

If a mathematical expression of B as a function of time is not readily available the approximation methods of time series can be used.[8,9] In any case the approximation is fast and convenient. The following example illustrates the method. If the flux density is considered to change linearly with time, (2) becomes

$$(4) \qquad \delta(t) = \alpha k^2 t^2$$

Time intervals of one-half microsecond, represented by Δt, are used with the time equivalent of the transducer length equal to $2 \Delta t$ and αk^2 equal to unity. The strain response to a unit step of flux applied to the medium may be written

$$(5) \qquad \Delta \delta = 1 + x + x^2$$

In this notation the coefficients of the terms x^0, x^1, x^2, ... x^n are equal respectively to the values of δ at times 0, v, $2v$, ... nv. The superposition for T equal to $3\Delta t$ is performed as a polynomial summation

$$(6) \qquad (t) = \frac{\begin{matrix} 1+ & x+ & x^2 \\ & 4x+ & 4x^2+4x^3 \\ & & 9x^2+ 9x^3+ 9x^4 \\ & & 16x^3+16x^4+16x^5 \end{matrix}}{1+5x+14x^2+29x^3+25x^4+16x^5}$$

It will be noted that one is not limited to easily expressed relations between B and t. Empirical data or oscillographic traces showing this relation are easily used to determine $\delta(t)$. Equation 6 represents a function with ordinates 1, 5, 14, 29, etc. at the times $0, v, 2v, 3v$ etc. as plotted in Fig. 2. This function in time corresponds to an inverted space function, since the first element in time, located at the origin, corresponds to the element of the mechanical pulse located farthest from the origin in space representation. The pulse shape in space then may be described as an inversion of (6)

$$(7) \qquad \delta(1) = 16x+25x^2+29x^3+14x^4+5x^5+x^6$$

The accuracy of the approximation is improved by reducing the time interval Δt. In the limit as Δt approaches zero the solution is exact.

III. THE OUTPUT OF THE RECEIVING COIL

For a given delay medium the four principal factors involved in the shape of the electric output signal are: the effective length of the receiver coil, the magnetization levels in the medium, the shape of the mechanical signal received, and the number of turns in the coil. There must be provided a magnetic bias

direction of the bias field is reversed the polarity of the output is changed. Additional uses due to this can be named. Information in binary code can be easily stored and read in the polarity of the magnetic bias field of a number of receivers. The reading is done without destruction of the stored information by the sonic pulse sent down the delay line, each bit of information being read through the output of its receiver.[5,6] Changes in the stored information are made by a change of bias field which is remanent magnetism in the line. This may be done by a pulse through the receiver. The same system with fixed bias fields may be used as word generators which supply, for example, constants used repeatedly.[5] By using reshaping equipment a circulating memory can be assembled, with the information, supplied as positive or negative pulses, circulating down the line and through the restoring system and back down the line until required in the computation.[5]

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MAGNETOSTRICTION FREQUENCY-CONTROL UNITS AND OSCILLATOR CIRCUITS

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Abstract. — Magnetostriction oscillators and associated circuits have been investigated as a means of frequency control in the 50 to 5000 kilocycle frequency range. Factors that affect stability are discussed. It is shown that the temperature coefficient of frequency of the resonator can be controlled by special heat treatment of a commercially available alloy, Ni-Span C. Frequency stabilities of ± 0.01 percent over the temperature range from $-55°$ to $+90°C$ have been achieved for resonators oscillating in the width-extensional and width-shear modes at frequencies between 250 and 2000 kilocycles. Maximum power capability of magnetostriction resonators is discussed and it is shown that these units may be limited either by heating or by exceeding the maximum strain of the material. Magnetostriction resonator units have been developed that cover the frequency range from 250 to 2000 kilocycles. For frequencies below 1000 kilocycles the units fit into a FT249 holder. The high frequency unit is 7/8 in. in diameter and one in. high and covers the frequency range above one Mc. These units all use resonator elements made from 0.010 in. flat stock material, have two coil coupling systems, and contain a permanent magnet to supply the required polarizing field.

I. INTRODUCTION

Magnetostriction oscillators were first considered for frequency control by G. W. Pierce[1] and associates [2,3] at Harvard University more than twenty-five years ago. These experimenters used magnetostrictively driven rods vibrating longitudinally at frequencies up to 300 kilocycles. A few years later work on magnetostriction oscillators was in progress in Europe.[4,5] F.D. Smith[6] used magnetostrictive rings wound with toroidal coils, producing circumferential vibrations, to stabilize the frequency of oscillator circuits.

While magnetostriction transducers continued to be investigated, particularly for ultrasonic and underwater sound systems, little attention was given to their further development as frequency control elements until the early part of World War II. At that time the supply of quartz for radio frequency control was scarce and metallic substitutes were investigated for this application by R. Adler [7,8] of Zenith Radio Corporation. By utilizing the intermediate dimension instead of the longest as the frequency determining dimension he was able to extend the upper usable frequency limit of these metallic resonator elements. Adler's resonator elements were made of flat stock material, usually in the shape of thin washers. Electromechanical coupling was provided by flat coils placed near the large surface of the resonator.

Probably the most serious limitation of magnetostriction oscillators was frequency instability produced by temperature changes. Even the earliest workers[9] in this field recognized this problem and searched for alloys with low thermal expansion and thermoelastic coefficients that could be used for resonator elements. An alloy known as Elinvar, extensively used for watch hair springs, had been found useful in reducing the large temperature coefficients of frequency measured for most magnetostrictive materials. To achieve favorable frequency stability characteristics with this material its composition had to be controlled to a degree of accuracy almost impossible to attain.

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During the course of this development a commercial alloy, Ni-Span C, has been adapted for use in resonator elements. This material, an age hardening alloy, can be controlled by heat treatment to produce either positive, negative, or near zero temperature coefficients of frequency; its application to magnetostriction resonators is described. Physical and electrical characteristics of magnetostriction resonators, the development of suitable magnetostriction holder units for the 250 to 2000 kilocycle frequency range and applicable use oscillator circuits are covered in this paper.

II. PHYSICAL AND ELECTRICAL CHARACTERISTICS OF MAGNETOSTRICTION RESONATORS

Modes of Oscillation and Methods of Excitation

The frequency of vibration of a resonant element is a function of its density, the elastic or shear moduli of the material, and the physical dimensions of the element. When the mode of vibration is longitudinal, the frequency of a half-wave metallic resonator element is given by the formula

$$(1) \qquad f = \frac{1}{\lambda}\sqrt{\frac{E}{\rho}} \approx \frac{10^5}{\ell} \quad \text{cycles per second,}$$

where λ is the wave length of sound in the material, ℓ is the length of the frequency dimension in inches, ρ the density of the material, and E its elastic modulus. For the shear modes of vibration the resonant frequency is about 60 percent of that given by (1). This results from the fact that the shear modulus is only about one-third the value of the elastic modulus for the metals that can be used for this application.

In determining the most desirable mode for a given frequency range several factors must be taken into account. The most important consideration is that the mode of vibration and method of excitation must yield good oscillating efficiency over the intended frequency range. The size of the resonator element and associated unit (which includes the magnetic polarizing field and electromechanical coupling system) should be as compact as possible. While the cost of the resonator material is insignificant compared to the other components of the magnetostriction unit, the fabrication of this element should be simple, preferably by cutting or blanking rather than a precision machining operation.

For frequencies up to a few hundred kilocycles half-wave longitudinal mode rods have been used extensively. These rods are generally clamped at the nodal point in their center. It is important that the clamp be precisely positioned, for otherwise considerable vibrational energy will be lost in the support. A magnetic polarizing field is applied longitudinally. Energization of the element is accomplished through electromechanical coupling coils wound about the rod. When an alternating voltage at the mechanical resonant frequency is applied to the coils, an alternating field is set up through the rod causing it to vibrate because of the magnetostriction effect. The magnetic bias is necessary since magnetostrictive elongation of the rod will occur with either polarity of field; thus without it, vibrations would tend to take place at twice the excitation frequency. The employment of the proper value of polarizing field is an important factor for obtaining optimum motional impedance, mechanical Q and fre-

quency-temperature characteristics of resonator units. These factors are considered in some detail in this paper.

Variations in resonator geometry are possible at these low frequencies. For example, an element that has a tubular form requires a lower polarizing field and generally exhibits a higher oscillating efficiency. Rectangular elements fabricated from flat-stock material have also been used at these frequencies with success. These strip elements lend themselves to easy fabrication.

Ring resonators may also be used to cover the frequency range below 250 kilocycles. By utilizing the circumference of the ring as the frequency determining dimension, the resonator unit can be kept fairly compact even at low frequencies. This mode of oscillation can be achieved by employing toroidal coils wound about the ring to obtain the necessary circumferential alternating and polarizing fields. This method has the disadvantage of making it impossible to remove the resonator from the coupling coils (i.e., for frequency adjustment).
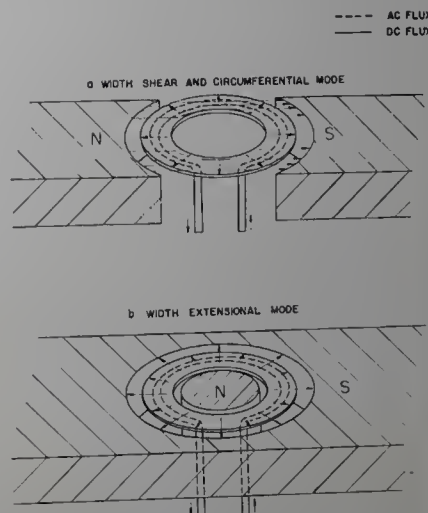

Fig. 1—Modes of oscillation.

To overcome this difficulty certain modifications in polarizing and coupling technique have been employed. The washer shaped resonator may be magnetized in two halves along its circumference as shown in Fig. 1(a). Coupling is accomplished by coils wound near the flat surfaces of the resonator. Alternating current in these coils produces circumferential as well as radial expansion and contraction because of the transverse magnetostriction effect. It is the circumferential component of motion that produces the desired mode of oscillation. To obtain high oscillating efficiency for this mode it is necessary to use resonators with large width factors so that the amount of circumferential motion will be large.

For frequencies above 250 kilocycles washer shaped resonator elements have proven most satisfactory. The intermediate dimension of these resonators is used in extension and shear to cover these higher frequencies. By utilizing these modes flat coupling circular coils can be placed close to the large surfaces of the resonator and good coupling efficiency may be attained.

Frequencies between 250 and 1000 kilocycles are best controlled by use of the width shear mode. The frequency dimension, which is the difference in radii of the washer, ranges in size from 0.3 in. down to 0.06 in. to cover this range, and the diameter of the element can be made less than one inch even at the lowest frequency. Excitation for this mode and employment of magnetic polarization is identical to that employed in the low frequency circumferential mode shown in Fig. 1(a). It is difficult to obtain a field that is completely circular; therefore a shear mode with circular symmetry is not considered practical. The component of alternating field in the radial direction through the resonator, in combination with the polarizing field, produces shear vibrations. A selection between the width shear and the circumferential mode can be made by resonator orientation since the resonator exhibits optimum magnetic and mechanical properties along the direction of rolling of the material.

At frequencies above 1000 kilocycles the width extensional mode shown diagramically in Fig. 1(b) proves most useful. In this mode a half-wave element is approximately 0.1 in. long at one Mc. At frequencies above two Mc harmonics of this mode show the best oscillating efficiency. A counter-wound coil can be used to produce an alternating field that adds to the polarizing field along one half of the frequency dimension of the resonator while subtracting in the other half to set up the full wave motion. Frequencies as high as five Mc have been controlled with full wave ring resonators.

An important consideration in the excitation of magnetostriction resonators is the employment of an efficient coupling system. The use of washer shaped elements allows flat coils to be placed close to the large moving surfaces to attain good oscillating efficiency at the higher frequencies.

Resonator thickness is another factor that affects oscillating efficiency. The depth of current penetration varies inversely with the square root of frequency. For thick resonators, a large portion of the element is not excited and is consequently carried along in motion by the rest of the resonator. While this consideration indicates the desirability for thin resonators, even at low frequencies, they must be made thick enough to prevent bending and warping. The result of an investigation to determine optimum resonator thickness has been a compromise between these two factors, and a thickness of 0.010 in. has generally been adopted for all frequencies.

Equivalent Electrical Circuit

When a magnetostriction resonator is energized by an alternating current through an electromechanical coupling coil, a counter-voltage caused by the motion of the resonator element is reflected into the coil. At the mechanical resonant frequency the motion of the resonator will be large, and consequently the reflected voltage, called the motional emf, will also be large. This voltage may be represented as an impedance reflected into the coupling coil that adds to the static value of the coil impedance. The equivalent electrical circuit representing this impedance is the inverse circuit representing a piezoelectric oscillator. If a comparison to crystal oscillators is made it is noticed that the active branch of the crystal, which may be represented by a series RLC circuit, is replaced by a parallel RLC network in the magnetostriction circuit. A static capacity paralleling the active branch of the crystal takes the form of a series inductance in the magnetostriction oscillator. This is the static inductance of

the electromechanical coupling coil. The impedance of the magnetostriction network then may be expressed by the equation

(2)
$$Z = R_c + jX_{L_c} + \frac{RX_LX_c}{X_LX_c + jR(X_L - X_c)}$$

The first two terms represent the coupling coil resistance and reactance while the motional parameters are given by the last term. At the anti-resonant frequency of the resonator the motional term of (2) appears as a pure resistance equal to R. This is the maximum value of impedance that the resonator can reflect into the coupling coil, and it occurs at the frequency where the resonator vibrational amplitude is a maximum.
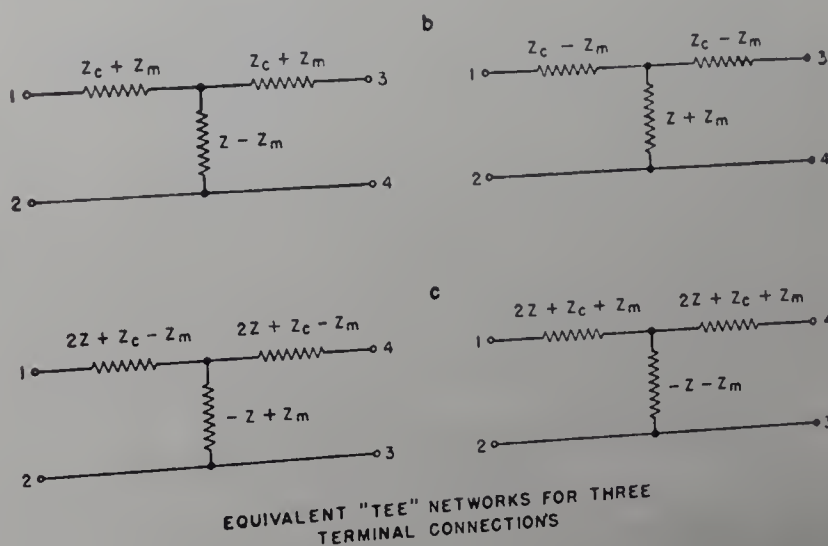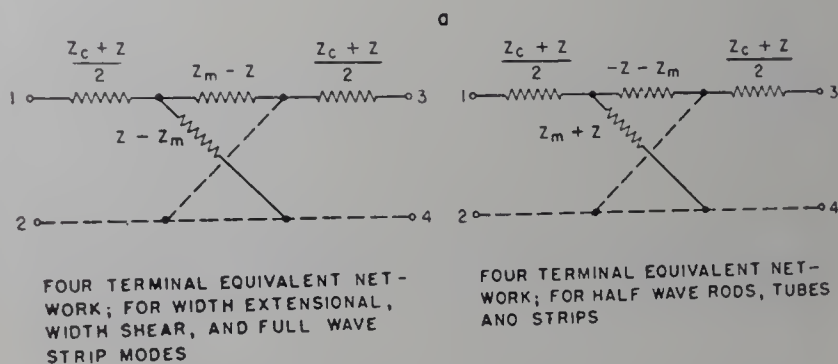
In practical resonator units two coils are generally used to couple electrical and mechanical energy to and from the resonator. This coil arrangement permits the use of the unit as a four terminal network or as an efficient two terminal element when the coils are connected in series. Four terminal equivalent circuits are shown in Fig. 2(a). It is noticed that the sign of the mutual impedance term Z_m is not the same for all modes of oscillation. In the half-wave longitudinal rod the coils must be connected with their fields aiding to produce motion in the resonator, while in the modes where coils are wound on either side of the washer shaped resonator their fields must oppose to obtain the desired oscillations.

Fig. 2—Two and four terminal equivalent circuits.
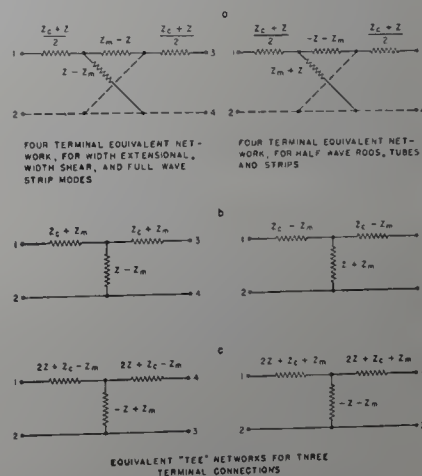
The four terminal network may be reduced to three by connecting terminals 2-3 or 2-4 together. The "T" equivalents are shown in Figs. 2(b) and 2(c). It is noticed that when the unit is used as a two terminal element the impedance between terminals 1-3 yields no motional impedance when terminals 2-4 are connected together. The proper coil connection is terminals 2-3, and the resulting impedance between terminals 1-4 is $4Z + 2Z_c \pm 2Z_m$, the sign of the last term depending upon the mode of oscillation. The coupling coil resistance and inductance are doubled for the series coil connection, while the motional impedance is increased by a factor of four, since this term is a function of the square of the number of electromechanical coupling coil turns.

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a

$\frac{Z_c + Z}{2}$ $Z_m - Z$ $\frac{Z_c + Z}{2}$ $\frac{Z_c + Z}{2}$ $-Z - Z_m$ $\frac{Z_c + Z}{2}$

$Z - Z_m$ $Z_m + Z$

FOUR TERMINAL EQUIVALENT NET-
WORK; FOR WIDTH EXTENSIONAL,
WIDTH SHEAR, AND FULL WAVE
STRIP MODES

FOUR TERMINAL EQUIVALENT NET-
WORK; FOR HALF WAVE RODS, TUBES
AND STRIPS

b

$Z_c + Z_m$ $Z_c + Z_m$ $Z_c - Z_m$ $Z_c - Z_m$

$Z - Z_m$ $Z + Z_m$

c

$2Z + Z_c - Z_m$ $2Z + Z_c - Z_m$ $2Z + Z_c + Z_m$ $2Z + Z_c + Z_m$

$-Z + Z_m$ $-Z - Z_m$

EQUIVALENT "TEE" NETWORKS FOR THREE
TERMINAL CONNECTIONS

...e reduced to three by connecting terminals
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...connection is terminals 2-3, and the result-

Stability Factors

When a magnetostriction resonator unit is used to stabilize the frequency of an oscillator circuit, components of the circuit will have an effect on frequency stability. Changes in reactance of the circuit elements and variations in tube characteristics due to voltage changes and aging contribute to circuit instability. As in the case of crystal stabilized circuits the stabilizing element should have a steep reactance characteristic at the control frequency in order to minimize the effect of circuit component variations. The steepness of the reactance characteristic at anti-resonance, expressed as the rate of change of reactance of the resonator unit with respect to frequency, is given by the equation

$$(3) \qquad \frac{dX}{d\omega_0} \cdot \frac{\omega_0}{X} = - \frac{2RQ}{\omega_0 L_c}$$

The significance of this equation can readily be seen from the motional impedance circle diagram of Fig. 3. The ratio of the circle diameter (which represents the motional impedance of the resonator) to the coupling coil reactance, $\omega_0 L_c$, should be as large as possible for a high stability coefficient.

Another important ratio that can be determined from the circle diagram is ratio of motional impedance to coil resistance, previously referred to as oscillating efficiency. Because coupling to the resonator element is inductive with an appreciable loss component for magnetostriction units rather than capacitive as in the case of crystals, this ratio must be considered in the design of magnetostriction oscillator circuits. If the oscillating efficiency is not large, it is difficult to design a circuit that will be assured of control by the resonator element. In particular, it becomes increasingly difficult to design an oscillator circuit that will oscillate only when the resonator is controlling the frequency of the circuit.
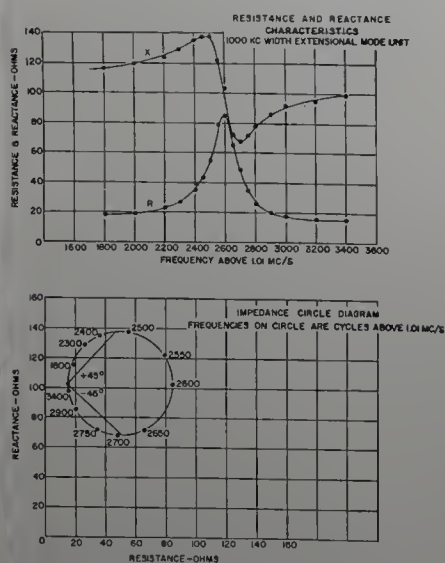

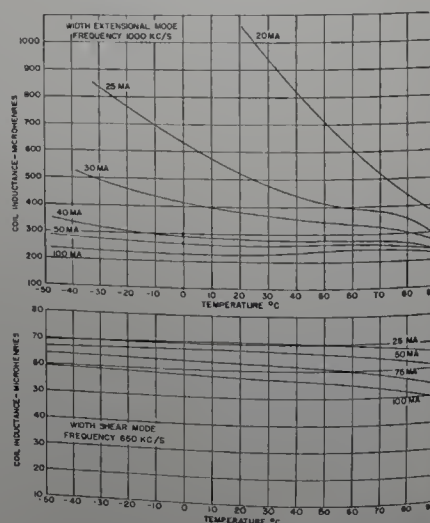Fig. 3—*Impedance characteristics of magnetostriction resonators.*

Fig. 4—*Variations in coupling coil inductance as a function of temperature and polarizing field.*

In crystal frequency control units, the Q factor of the resonator must exceed twice the ratio of the static to active capacity if the reactance characteristic is to become positive at any frequency. This criterion must be satisfied if the crystal is to be used in oscillator circuits without inductive compensation. In magnetostriction units, a similar relationship exists between the resonator Q, the equivalent inductance of the resonator and the static inductance of the coupling coil. It is apparent from the motional impedance circle diagram shown in Fig. 3 that the Q of the resonator must exceed twice the static to active inductance ratio of the resonator unit if the reactance characteristic is to become negative at any frequency.

Temperature stability over a wide temperature range is an important consideration in the development of practical frequency control devices. Temperature stability of the resonator element is achieved by employment of an iron-nickel-chromium-titanium alloy, Ni-Span C, and subjecting it to a special metallurgical processing cycle. The metallurgy of the resonator alloy will not be discussed in any great detail here, but enough information will be given to provide an appreciation of the temperature stability problem encountered in this development. In addition to achieving a flat frequency-temperature characteristic the metallurgical processing produces high Q resonator elements. Values of Q range from 2,000 in the 50 to 100 kilocycle range to almost 10,000 in the Mc range.

Temperature also affects the coupling coil inductance of resonator units. This occurs because the resonators must necessarily be closely coupled to the electromechanical coils to achieve a high oscillating efficiency. The permeability of the resonator material is greatly affected by temperature, for it is the low Curie point of the metal that maintains its constant frequency characteristics over a wide temperature range. Variations in permeability of the resonator change the coupling coil inductance and consequently introduce instabilities into the associated oscillator circuit. There are several methods of remedying or compensating for the temperature coefficient of the coupling coil inductance. These methods included the use of magnetic compensating elements and electrical networks. A careful study of the methods of compensation indicate that the most satisfactory solution of this problem is to operate the magnetostriction unit at a polarizing flux density where the inductance variation is the least. Figure 4 shows the variation in coil inductance as a function of temperature and polarizing field. The values of field strength are represented in the figure by polarizing current for the particular test holders used. Magnetic saturation of the resonators is approached at polarizing currents in the vicinity of 100 ma. For the width extensional mode the inductance variation becomes less as the polarizing field strength is increased. When choosing a value of field strength that will yield a small coil inductance variation, the proper heat treatment must be selected to produce a low frequency-temperature coefficient for the resonator.

In the shear mode the coil inductance variation is relatively small at all values of polarizing field. The magnetic field configurations offer an explanation for the radically different behavior of coil inductance characteristics observed for the two modes. In the extensional modes the polarizing and alternating fields are parallel while in the shear modes they are aperpendicular; consequently the polarizing field will have a larger effect on incremental permeability and hence coil inductance in the former case.

Power Capability of Resonator Units

There are two factors that may limit the maximum power dissipation in a magnetostrictive resonator; the maximum strain it can withstand without permanent yielding, and the heating effect due to the power dissipation. The maximum power dissipation in terms of maximum strain can readily be determined from the equations of motion of the resonator.

The velocity distribution in an unclamped resonator element is given by the equation

$$(4) \qquad v = V \cos \frac{2\pi x}{\lambda}$$

where x is measured from one end of the resonator, λ is the wavelength of sound in the resonator material.

The stored energy $\mathcal{E}_s$ is equal to the sum of the average kinetic and average potential energies, or twice the average kinetic energy. The average kinetic energy density is

$$(5) \qquad \tfrac{1}{2}\, \rho(\bar{v})^2$$

where ρ is the density of the resonator material.

$$(6) \qquad \tfrac{1}{2}\,\rho(\bar{v})^2 = \tfrac{1}{4}\,\rho V^2 \cos^2 \frac{2\pi x}{\lambda}$$

The stored energy per unit area is

$$(7) \qquad \tfrac{1}{2}\,\rho\, V^2 \int_0^{\frac{n\lambda}{2}} \cos^2 \frac{2\pi x}{\lambda}\, dx = \frac{n\lambda}{8}\,\rho V^2$$

where n is the order of harmonic of the resonator. The total stored energy in a resonator is therefore

$$(8) \qquad \mathcal{E}_s = \frac{n\lambda A}{8}\,\rho V^2$$

where A is the cross-sectional area of the resonator element normal to the frequency dimension. Equation (4) can now be written

$$(9) \qquad v = \sqrt{\frac{8\mathcal{E}_s}{n\lambda\rho A}}\, \cos \frac{2\pi x}{\lambda}$$

The velocity is also a sinusoidal function of time

$$(10) \qquad v(x,t) = \sqrt{\frac{8\mathcal{E}_s}{n\lambda\rho A}}\, \cos \frac{2\pi x}{\lambda}\, \epsilon^{j\omega t}$$

and the displacement at any time is

$$(11) \qquad d(x,t) = \int v\, dt = \frac{1}{j\omega} \sqrt{\frac{8\mathcal{E}_s}{n\lambda\rho A}}\, \cos \frac{2\pi x}{\lambda}\, \epsilon^{j\omega t}$$

The strain in the resonator is given by the derivative of the displacement.

$$(12) \qquad s = \frac{\partial d}{\partial x} = \frac{j\pi}{\omega\lambda}\sqrt{\frac{32\mathcal{E}_s}{n\lambda\rho A}}\ \sin\frac{2\pi J}{\lambda}\ \epsilon^{j\omega t}$$

The maximum strain S in the resonator occurs at $x = \dfrac{(2n-1)\ \lambda}{4}$ and is equal in magnitude to

$$(13) \qquad S = \frac{\pi}{\omega\lambda}\sqrt{\frac{32\mathcal{E}_s}{n\lambda\rho A}}$$

The average stored energy can be expressed as a function of the power dissipated in the resonator. The power dissipated is the product of the energy dissipation per cycle and the frequency.

$$(14) \qquad P = \mathcal{E}_d\ (\text{per cycle})\cdot f = \frac{\omega\mathcal{E}_s}{Q} = \frac{\omega^3\ \lambda^3\ n\rho A\ S^2}{32\pi^2 Q}$$

Using the relation $\omega = \dfrac{2\pi}{\lambda}\sqrt{\dfrac{E}{\rho}}$, where E is the elastic modulus of the resonator material, and multiplying by 10^{-7} to express the power in watts and other quantities in c.g.s. units, we have

$$(15) \qquad P = \frac{n\pi\ E^{3/2}\ A\ S^2}{4\rho^{1/2}\ Q\ 10^7}\ \text{watts}$$

The maximum power capability of Ni-Span C resonator material can now be evaluated in terms of the mechanical properties of the material. This material, when aged at 900°F, has an elastic modulus of 1.55×10^{12} dynes/cm², a density of 8.0 gr/cm³, a maximum strain within the elastic limit of 10^{-3}, and a Q of approximately 8000. Substituting these values into (15) we have

$$(16) \qquad P = 6.7\ nA\ \text{watts}$$

Standard ring resonators oscillating as half wave extensional elements at a frequency of 1 Mc per second have an average area of 7.5×10^{-2} cm² and a power capacity of 0.5 watt before the elastic limit of the material is reached.

Figure 5(a) shows the effect of level of excitation on the resonant frequency of one of these resonators. It is noticed that the frequency increases rapidly at power levels above 0.2 or 0.3 watt. This is the same effect observed for this resonator during a temperature run (at low levels of excitation) at approximately 120°C as the Curie point of the alloy is approached. The maximum strain in this resonator could not be reached because the Q decreases rapidly as the resonator becomes hot and the amplitude of oscillation diminishes.

Figure 5(b) shows frequency-power characteristics for a resonator only 0.005 in. thick. Since the rate of dissipation of heat from the resonator is a function of its surface area, heating occurs at approximately the same power level, but the maximum strain power level is decreased for thin resonators since it is proportional to the resonator thickness. The sharp decrease in frequency near 0.3 watt is attributed to stressing beyond the elastic limit. A permanent reduction in resonant frequency is measured when the power level is again reduced. This permanent change in frequency must be attributed to a physical change in the resonator.
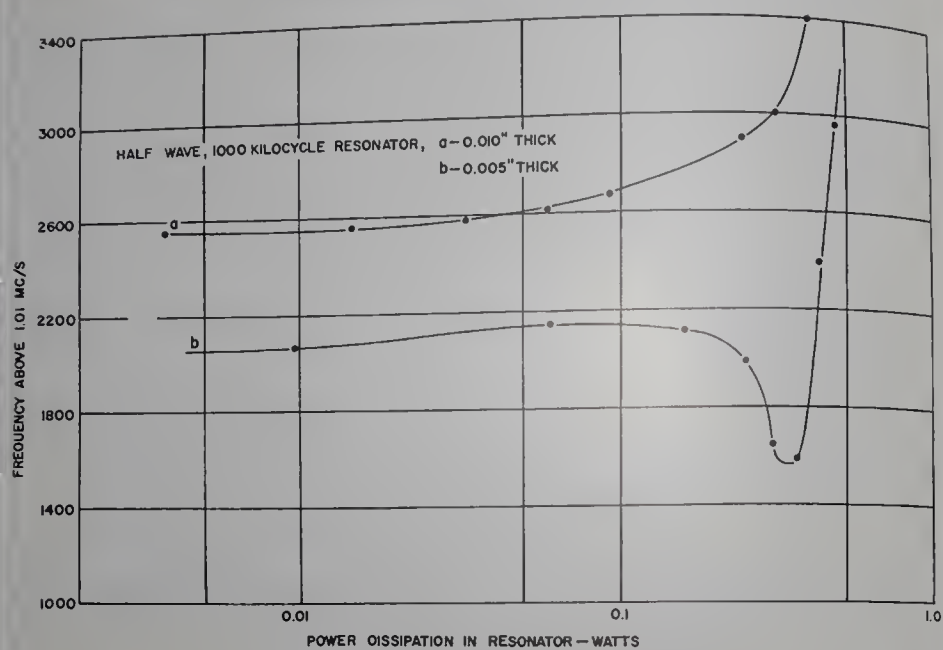
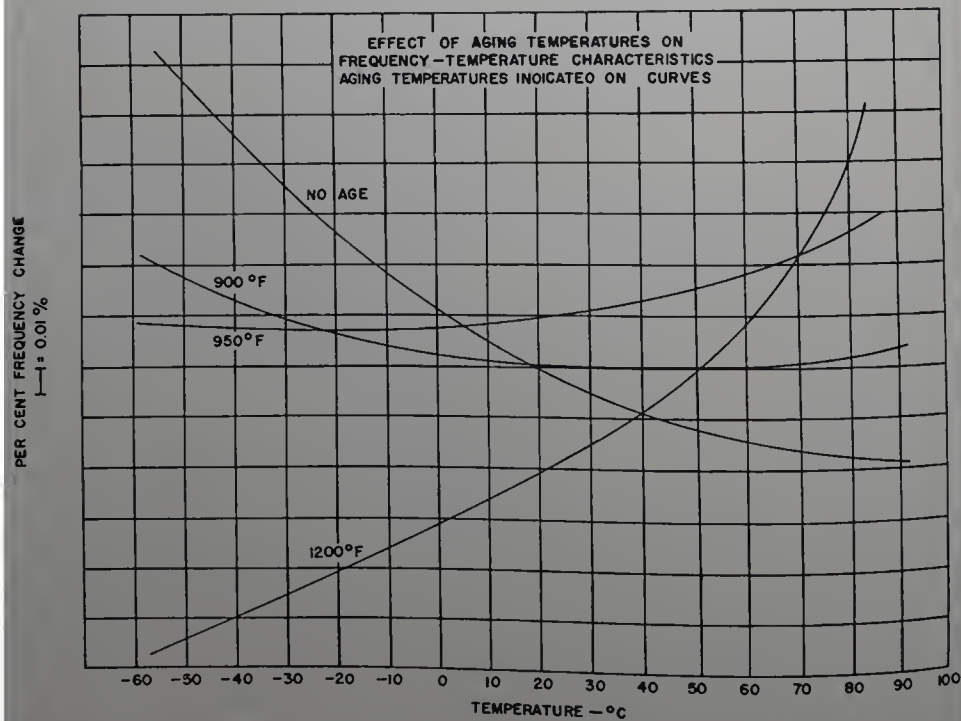Fig. 5 —*Effect of level of excitation on resonator frequency.*

Fig. 6 — *Effect of aging temperatures on frequency-temperature characteristics.*

III. METALLURGICAL PROCESSING OF MAGNETOSTRICTION RESONATORS

Effect of Resonator Composition on Performance

Ni-Span C, an iron-nickel-chromium-titanium alloy, was chosen as the resonator material because its temperature coefficient of frequency can be altered by heat treatment. The extent of the change caused by heat treatment is determined by the chemical composition of the alloy, and heats which fall outside of a definite analysis range cannot be used for resonators which must retain a flat frequency-temperature characteristic.

The specific heat treatment consists essentially of a solution treatment followed by an aging treatment. The solution treatment, carried out at 1750°F, dissolves a nickel-titanium compound in the matrix of iron-nickel to form a solid solution. A quench following this treatment serves to retain the compound in a super-saturated solution. The aging treatment is carried out at a lower temperature (800-1300°F), and serves to regulate the thermo-elastic coefficient by precipitating a controlled amount of compound out of solution. This also has a hardening effect on the alloy. The effect of aging temperatures on frequency-temperature characteristics of extensional mode resonators is shown in Fig. 6.

Some of the constituent elements of Ni-Span C also have another potent effect, that of lowering the Curie point, or magnetic transformation temperatures of the alloy. Chromium is particularly potent in this regard and heats with high chromium content may be unusable for this application. As the high temperature end of the frequency-temperature plot is reached the frequency-temperature characteristics suddenly increase, for their physical properties undergo a marked change in the vicinity of the Curie point. Heats with high chromium content show this rise at a much lower temperature than a normal heat and thus lose their favorable flat frequency-temperature characteristics.

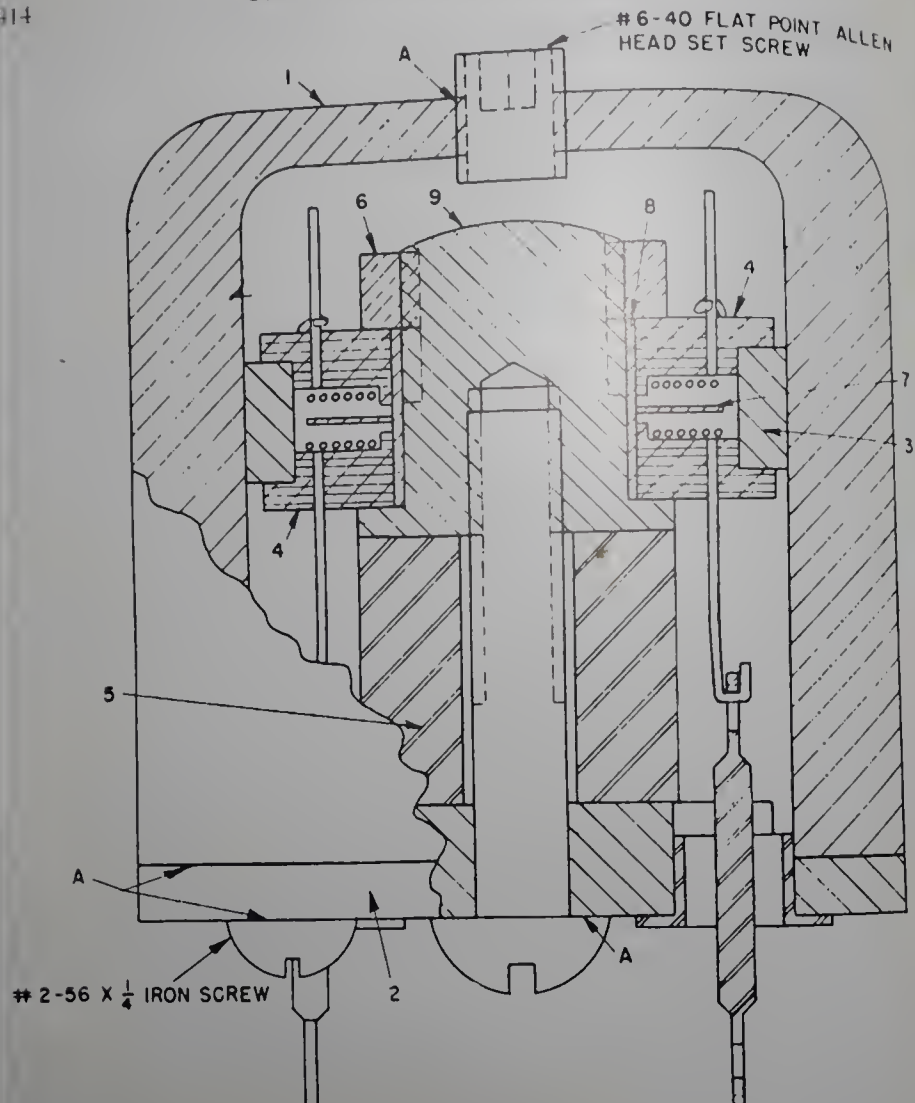## Heat Treatment for the Various Modes of Oscillation

Separate heat treatments are required for each mode of oscillation. Strips, rods, and tubes (all shapes used for low frequency resonator elements) should be heat treated in the same cycle as the other extensional mode elements. All resonators whose frequency depends on the modulus of elasticity E may be heat treated similarly regardless of their fundamental frequency.

Elements for the 250-1000 kilocycle frequency range depend on the shear modulus R as a function of frequency. This modulus has a different temperature coefficient from the elastic modulus and a different response to heat treatment. Although the same solution treatment may be used, most heats require approximately 300°F higher aging temperatures to obtain optimum characteristics in the shear mode than in the extensional mode.

IV. MAGNETOSTRICTION HOLDER UNIT DEVELOPMENT

Width Extensional Mode Unit

During the early part of this research and development program the characteristics of the resonator elements were of primary interest. The holder units at that time were considered as test units, serving as a source of magnetic bias and providing a method of exciting the resonator. These are of course the primary functions of all magnetostriction holder units, but in the practical units efficiency and compactness are also of primary importance.

Fig. 7 Magnetostriction holder assembly for width extensional mode (1-2 Mc.).

In the design of a unit for frequencies above one Mc, the width extensional mode, using washer shaped elements, has been employed. An assembly drawing of a holder unit of this description is shown in Fig. 7. In the test holders polarizing bias was obtained electromagnetically. Considerable reduction in size of the unit can be achieved by employing a permanent magnet (5) for this
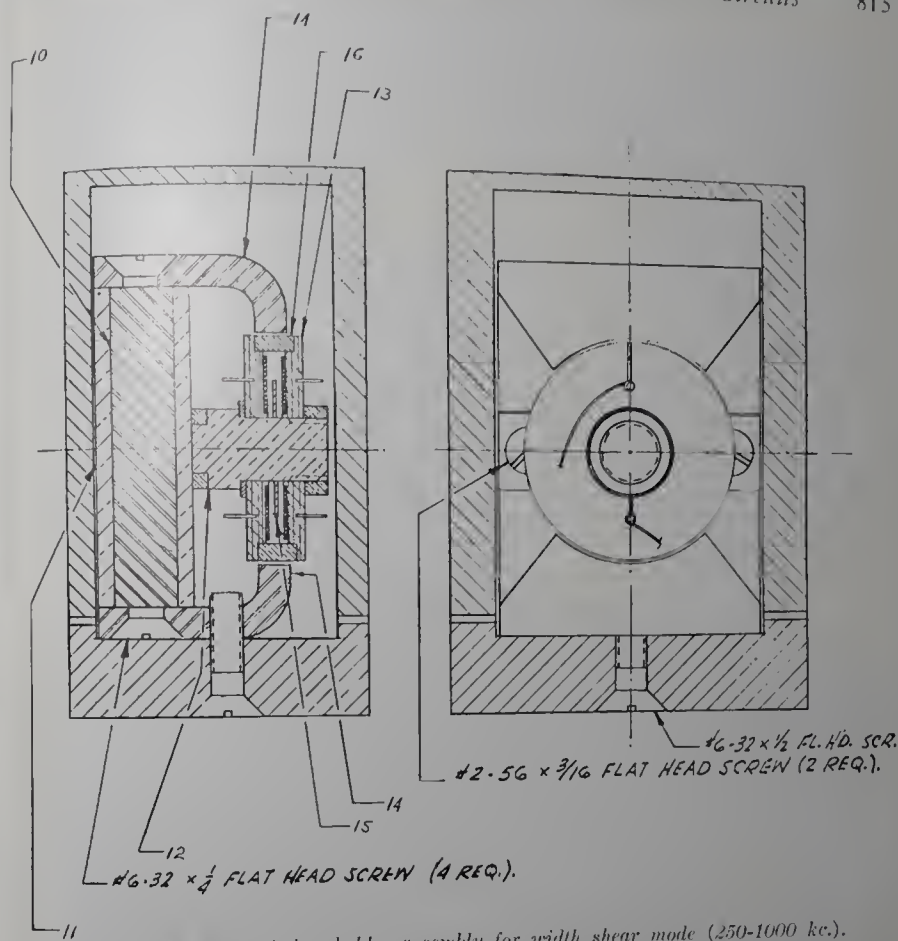
Fig. 8—*Magnetostriction holder assembly for width shear mode (250-1000 kc.).*

purpose, since in addition to saving winding space the need for a current supply is eliminated. The size of magnet was determined by calculating the amount of magnetomotive force necessary almost to saturate the resonator, allowing for the reluctance of the magnetic circuit including the air gaps. An iron shunting screw is provided for reducing the field through the resonator to the point of optimum performance. A shell of soft iron (1), base plate (2), a central pole piece (9), and outer spacer ring of the same material (3) complete the magnetic path through the resonator (7). The soft iron shell shields the unit magnetically and provides an easy means of hermetically sealing the entire unit.

The coupling assembly for this unit (4) consists of two flat coils wound on bakelite forms. The iron outer spacer ring provides the necessary spacing between coils to prevent the resonator from binding. A brass nut (6) holds this assembly in place. This ring and the brass inner sleeve (8) also serve as shorted turns to concentrate the alternating field of the coils in the resonator washer. All four coil leads are brought out through feed-through insulators in the base of the unit. The coils can be used separately to form three or four terminal

networks, or they can be used in series to increase the efficiency of the unit by a factor of two over that of a single coil when it is desired to use the unit as a two terminal element. When this connection is used, a motional impedance of almost 400 ohms results for 75 turn coils of no. 36 wire. This is a convenient level for oscillator circuits that employ the unit in the cathode circuit.

Width Shear Mode Unit

A permanent magnet magnetostriction unit that fits into a FT 249 crystal holder has been designed for the 250-1000 kilocycle range, Fig. 8. In this unit the polarizing field is supplied from an Alnico VI bar magnet (10) that fits between two brass channels (11) to form a base. A brass arbor (12) fastened to these channels centers the coil assembly (13) between the soft iron pole pieces (14). This coil assembly consists of the resonator element (15), two coupling coils, and a brass spacer ring (16). An inner sleeve is not necessary in this unit since the arbor that passes through the center of the resonator is non-magnetic and the resonator is not attracted to it. A small key is used to position the resonator so that proper gain orientation for maximum oscillating efficiency is attained.

V. OSCILLATOR CIRCUIT DEVELOPMENT

The design of magnetostriction oscillators calls for the employment of the magnetostriction unit as a two or four terminal element in a circuit that will sustain controlled oscillations. The problems of matching impedances and satisfying phase conditions for oscillation require the consideration of networks that couple the magnetostriction unit to the circuit.

Oscillators Employing the Magnetostriction Unit as a Two Terminal Element

The magnetostriction unit when considered by itself generally has a reactance characteristic that is positive at all frequencies. To use it as a frequency control element in an oscillator circuit where zero phase shift through this unit is required, it is necessary to cancel the static inductance of the coupling coil with other reactive elements. The simplest method of doing this is to series on parallel resonate the magnetostriction unit with a capacitance. In the series resonance case, the impedance of the network approaches a minimum near resonance, but at the mechanical resonant frequency of the resonator the impedance of the circuit increases sharply due to the motional impedance of the vibrating element. The parallel resonant method of inductance compensation produces a high impedance near mechanical resonance; the resonator motional impedance then decreases the network impedance sharply at mechanical resonance.

Application of this latter network to an oscillator circuit will be described. In a practical oscillator circuit oscillations should cease when the magnetostriction unit is removed or the element restricted from motion. The cathode coupled circuit shown in Fig. 9 requires that the feedback path between cathodes be less than a critical value, which is a function of cathode resistance of the circuit, if oscillations are to be maintained. When the magnetostriction unit is used in parallel with a resonating condenser as previously described and the cathode impedances are adjusted to produce conditions for oscillation only at mechanical resonance, the requirements for frequency control are then achieved. The exact
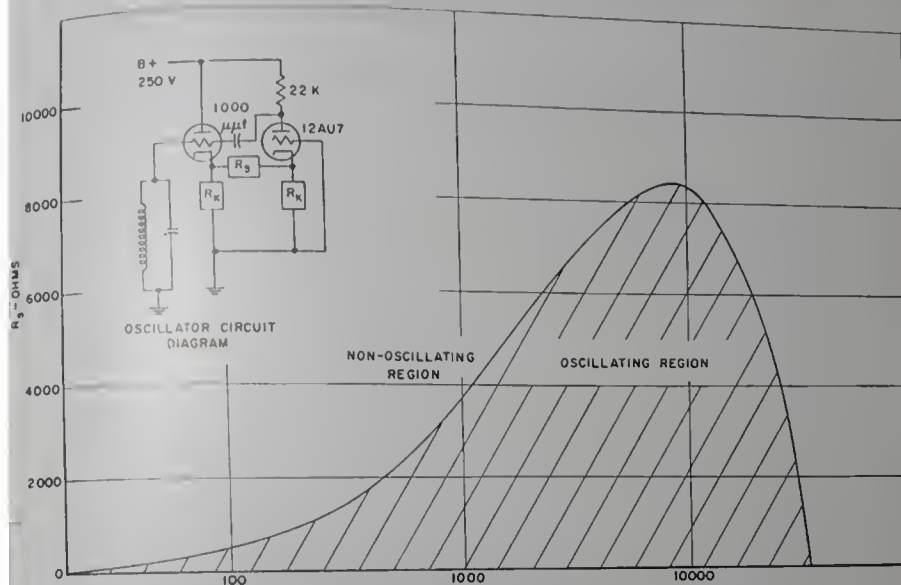
Fig. 9 — Oscillating characteristics for cathode coupled oscillator.

frequency of oscillation is then determined by the phase characteristics of the circuit. The phase of the parallel circuit comprising the coupling coil and condenser approaches zero at resonance. The phase characteristics of the magnetostriction element change rapidly from $+90°$ to $-90°$ at mechanical resonance. This rapid shift in phase with frequency produces highly stable oscillations. The impedance of a parallel combination of condenser and magnetostriction unit will drop below the critical value for oscillation at frequencies considerably above and below resonance; however the selectivity of the tuned grid circuit prevents oscillations at these frequencies.

A total frequency deviation of approximately 20 cycles is measured for the oscillator when operated over a temperature range from $-55°$ to $+90°$C. Other types of circuits employing the resonator unit as a two terminal element have been used successfully. These circuits include transformer coupled, modified Hartley, tuned grid, tuned plate-tuned grid, and cathode follower oscillators. Bridge networks (4 terminal) utilizing the magnetostriction unit as a two terminal element have also been devised.

Oscillators Employing the Magnetostriction Unit as a Four Terminal Element

Resonator units with low mutual coupling between coils will transfer energy only at the mechanical resonant frequency of the magnetostriction element; the associated circuit therefore can be stabilized at this frequency. Low frequency rod-type resonators can be designed with adequately shielded coils to meet this requirement. Circuits with one coil of the magnetostriction unit in the grid circuit and the other in the plate of a vacuum tube circuit have been used successfully (without tuning elements) to sustain and control oscillation.

VI. CONCLUSIONS

Magnetostriction resonator units show promise as a quartz substitute for r-f control. Advantages and shortcomings of the metallic substitute are listed below:

1. The upper frequency limit of practical magnetostriction oscillators (in the present state of development) is below three Mc. Best results have been achieved in the frequency range from 250 to 1000 kc, where low cost production quartz crystals show their poorest characteristics.

2. Magnetostriction resonators have lower Q values than well designed quartz units. This is not a serious limitation for in many applications ambient temperature changes will be the primary factor in determining frequency stability of the oscillator.

3. Because of the comparatively low oscillating efficiency of magnetostriction units oscillator circuits employing these units are more critical in adjustment than crystal oscillator circuits.

4. Frequency stabilities approaching those of quartz can be obtained for magnetostriction oscillators operating over a wide temperature range by simple metallurgical processing of resonator elements.

5. Magnetostriction resonator units can be designed for compactness. (A one Mc unit design is 7/8 in. in diameter and less than one in. long). The weight of these units is necessarily greater than that of quartz crystals because of the magnetic material required in the polarizing field.

6. Frequency adjustment is a relatively simple operation for magnetostriction resonators. Small amounts of material can be removed from the outer circumference of the resonator elements with abrasive material to increase the resonant frequency. The frequency can be adjusted downward by applying pressure to the element. Small eccentricities in the washer shaped elements do not appreciably affect their performance.

7. Magnetostriction units use a minimum of critical material.

8. The cost of magnetostriction units in production quantities would be low.

ACKNOWLEDGMENTS

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